

Homework 1

Due August 28th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. The book is <https://archive.org/details/complex-variables-2ed-dover-1999-fisher/page/n23/mode/2up>.

Do 1.1.1e, 1.1.2b, 1.1.4, 1.1.5g, 1.1.6b from page 9 of the book. (Note that some of the answers are on page 397.)

Also do the following problems:

1. (a) Solve for $\operatorname{Re} z$ and $\operatorname{Im} z$ in the system $z = \operatorname{Re} z + i \operatorname{Im} z$, $\bar{z} = \operatorname{Re} z - i \operatorname{Im} z$ to derive formulas for $\operatorname{Re} z$ and $\operatorname{Im} z$ in terms of z and \bar{z} .
(b) From the above answer, what can you conclude about where in the complex plane it is possible for $z + \bar{z}$ and $z - \bar{z}$ to fall? Illustrate by plotting a few examples.
2. Let $z = 1 + i$.
 - (a) Find $\operatorname{Arg} z$, find all values of $\arg z$, and find $|z|$.
 - (b) Find $\operatorname{Arg}(z^{10})$ and $|z^{10}|$.

Hint: Do not brutally expand $(1 + i)^{10}$! Use $|zw| = |z||w|$ and the corresponding rule for arguments.
 - (c) Use the above to simplify z^{10} .
 - (d) For which positive integers n do we have $\operatorname{Arg} z = \operatorname{Arg}(z^n)$? Find a formula for all of them and calculate the four smallest ones.