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MA 425/525
Fall 2023

Homework 1

Due August 30th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. The book is `https://archive.org/details/complex-variables-2ed-dover-1999-fisher/page/n23/mode/2up`. There are some hints on the next page.

- Nonbook problem: Solve for $\operatorname{Re} z$ and $\operatorname{Im} z$ in the system $z = \operatorname{Re} z + i \operatorname{Im} z$, $\bar{z} = \operatorname{Re} z - i \operatorname{Im} z$ to derive formulas for $\operatorname{Re} z$ and $\operatorname{Im} z$ in terms of z and $\bar{z}$.
- From Section 1.1 (page 9), numbers 1(e), 2(b), 4, 5(g), 6(b), and 15.
- From Section 1.2 (page 21), numbers 15 (for this one, also draw a sketch of the circle in the complex plane), 24 (for this one, write all solutions in the form $z = a + bi$ with a and b real), and 30.

Hints:

- For 1.1.15, use the nonbook problem at the beginning of the homework to show that $|z-w|^2 = |z|^2 - 2\,\mathrm{Re}(z\bar{w}) + |w|^2$ and use the definition of equilateral triangle. Note that there is an outline of a solution on page 397 but you should be sure to fill in all the missing steps carefully if you use it.
- For 1.2.15, recall that you can find the center of the circle passing through given points A, B, and C by finding the intersection point of the perpendicular bisectors of the segments AB, AC, and BC. In this example the perpendicular bisectors of the given points are nice.
- For 1.2.30, use the formula for $|z-w|^2$ from problem 1.1.15 to get a formula for $|z^2| - \mathrm{Re}(Az)$ in terms of $|z-w|^2$ and $|w|^2$ for a suitable w.